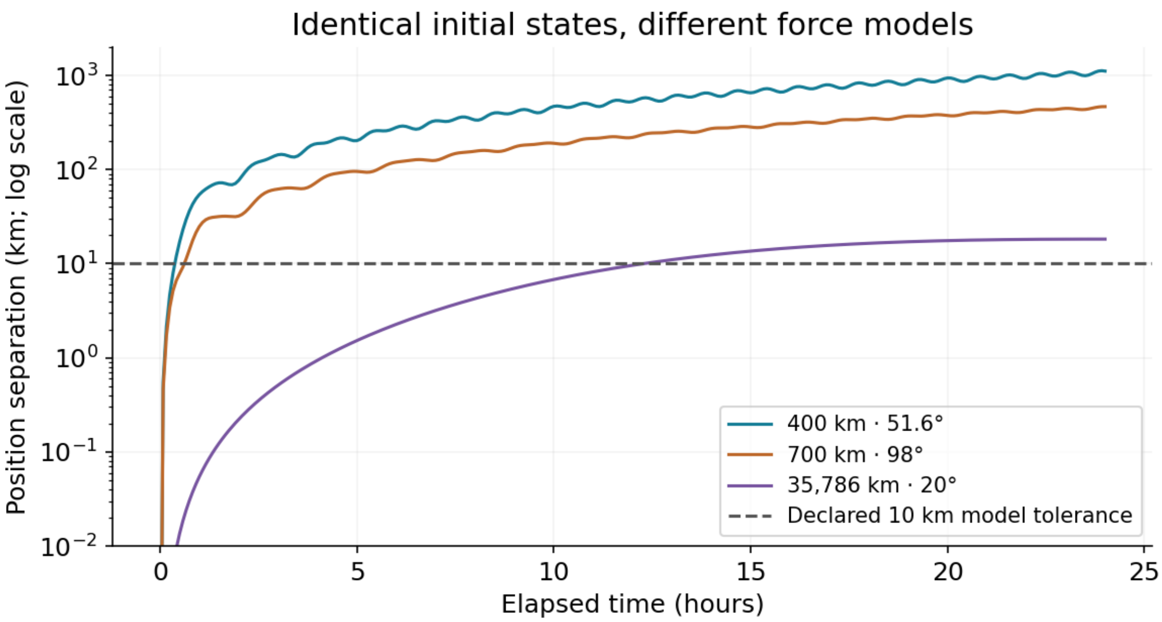
A QUESTION, AN INSTRUMENT, A TEST

When is two-body gravity good enough?

Orbital Sandbox is an interactive laboratory for changing a spacecraft's velocity, observing the resulting orbit, and examining the limits of a simplified model. This case study connects its origin, a reproducible maneuver, and a bounded numerical investigation.

The experiment

Start the same spacecraft in the same Earth-centered position and velocity. Propagate it for one day with point-mass gravity and again with Earth's J2 oblateness term. Declare a 10 km position-separation tolerance before evaluating the scenarios. Ask when the simpler prediction first exceeds it.



The two models separate even when their numerical implementations agree closely with independent checks. The 10 km tolerance is an illustrative positional requirement, not a mission-navigation standard.

Result: the first samples over 10 km occur at 25 minutes for a 400 km orbit, 40 minutes for a 700 km retrograde orbit, and 12 hours 20 minutes for a high Earth orbit. Samples are five minutes apart; these are brackets, not exact crossing times.

From Jupiter's moons to a sandbox

The original question

Dash first saw Jupiter's four largest moons through his great-grandfather's telescope, with two on each side of the planet. He wondered how often all four might appear on the same side. He calculated their orbital periods by hand, then found a NASA animation that helped him see their changing positions.

That question led to wanting to explore other real and hypothetical orbits: changing the central body, altitude, and velocity, then seeing how the motion changed. Kerbal Space Program was another influence. The project also became a way to try building with Codex.

How the work is made

The website uses AI-generated and AI-assisted code. Dash describes what he wants, tests working versions, questions results and visuals that do not make sense, and directs revisions. This technical package was assembled with coding-agent assistance. Its numerical claims are supported by the accompanying calculations; they are not claims that Dash independently wrote every algorithm or experimentally measured a real spacecraft trajectory.

Corrections became part of the project

Reference points: altitude is height above the surface; semi-major axis is measured from the body's center. Keeping those quantities separate prevents a plausible-looking but misleading readout.

Event times: a maneuver is reported at its scheduled mission time, even when accelerated playback skips across that instant between frames.

Model boundaries: a 3D picture alone does not establish 3D mission design. The current editor, state validation, RTN burns, saved plans and readouts preserve all three spatial dimensions.

The historical mission archive is a separate data product: it replays sourced JPL Horizons trajectories. It does not demonstrate that the editable two-body solver reproduces those missions. Private application essays supplied story context and are not included in this package.

One canonical state, two displays

The editable planner stores position r in kilometres, velocity v in kilometres per second, and mission time in seconds, in a body-centered inertial Cartesian frame. Earth uses gravitational parameter 398600.4418 km^3/s^2 and radius 6378.137 km. Surface texture orientation and spacecraft size are illustrative.

Coasts and impulses

Two-body acceleration is $-mu\ r / |r|^3$. A universal-variable Kepler solver propagates between instantaneous maneuvers across elliptical, parabolic and hyperbolic states. It uses a 1e-12 relative solve tolerance, a bounded iteration count, and subdivision for difficult steps. Non-convergence returns an explicit failure, not a guessed state.

An impulse changes velocity in the local radial/transverse/normal frame: $R = r/|r|$; $N = (r \times v)/|r \times v|$; $T = N \times R$. Positive N changes the orbital plane. A stored reference normal provides a deterministic convention when angular momentum is degenerate.

Readouts and editing

Specific energy is $|v|^2/2 - mu/|r|$. Specific angular momentum is $r \times v$. The same state supplies eccentricity, inclination, node, periapsis, apoapsis and period. Open conics have no finite apoapsis or period. Surface crossings and unreachable burns are explicit outcomes.

The editor accepts all six Cartesian components or six elliptical orbital elements. Circular and equatorial states need angle conventions because individual classical angles become undefined. Regression cases cover polar, retrograde and nearly equatorial states. Spatial plans use numeric editing; planar pointer editing remains available for planar states.

New links preserve 3D state and normal burns; old planar links and files are migrated with their original meaning. Both canvas and WebGL display canonical propagated positions. A 2D oblique view projects the third coordinate rather than discarding it.

The boundary

The free sandbox excludes drag, J2, other bodies, radiation pressure, finite thrust and enforced mass depletion. The separate investigation adds only J2. Neither is a complete mission-design or flight-safety tool.

A REPRODUCIBLE MANEUVER

Add 100 m/s. Predict the new orbit.

The 80-second tour begins in a circular Earth orbit at 400 km altitude. Circular speed is 7.668558 km/s and the period is 92.5604 minutes. At mission time zero, a transverse impulse adds 0.100 km/s. Radial and normal components remain zero.

Prediction before playback

Use circular speed v = sqrt(mu/r), then the vis-viva relation $1/a = 2/r - (v + 0.100)^2/mu$. Because this modest prograde impulse begins on a circular orbit, the burn point becomes periapsis and the opposite apsis radius is 2a - r.

This predicts an apoapsis altitude of **765.453282 km**. The canonical post-burn state gives **765.453282 km**. Their absolute difference is 2.728e-12 km. Position stays fixed during the impulse; velocity and specific energy change immediately.

What to watch

The orbit becomes elliptical. The spacecraft slows as it climbs toward apoapsis. The path and telemetry use the same propagated state. Energy drift is checked during the coast; the deliberate energy jump during the burn is not treated as a conservation failure.

The tour has five steps: the motivating question, a circular orbit, the velocity change, an analytical check, and the model-limit question. Start, pause, reset and individual steps are available. Reduced-motion mode keeps representative stationary positions, and the complete explanation remains available as text.

Continue the experiment

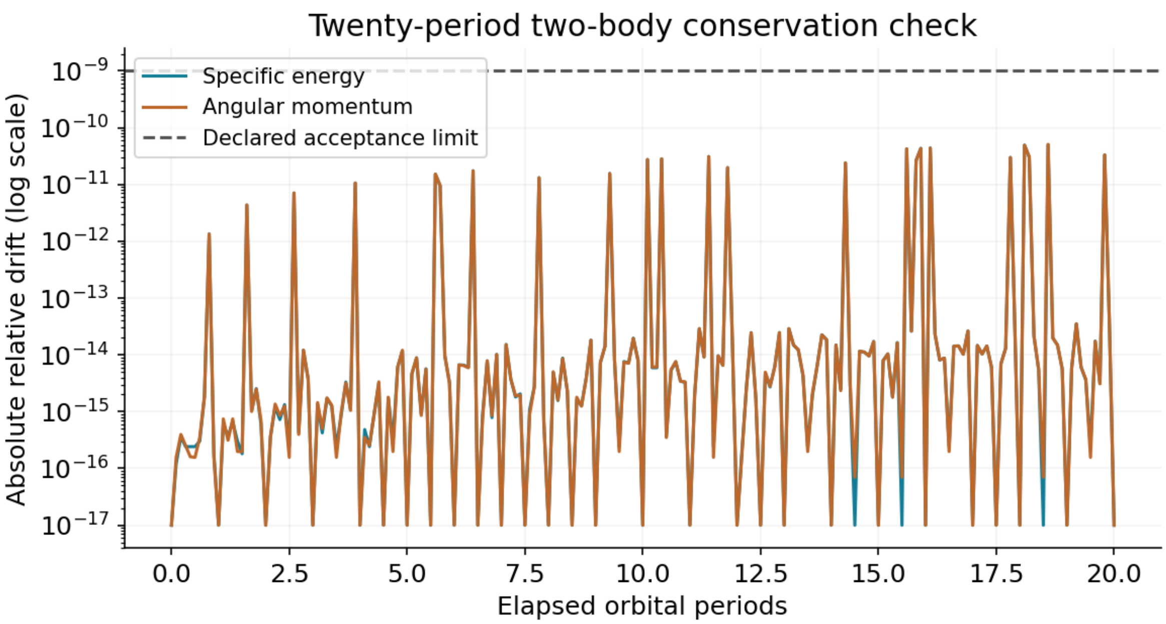
Open the example in a new sandbox tab to change the burn without replacing an existing plan. Try a normal component to change the plane, or a mixed impulse to see why transverse sign alone does not determine the energy change. A zero burn leaves the orbit unchanged. A sufficiently large impulse can remove the finite apoapsis entirely.

The companion 80-second film is generated from the same propagator at 12 frames per second. Its captions are the narration; there is no audio track. It is an explanatory rendering, not a recording of an observed mission.

Separate numerical error from model error

Five core cases cover circular return, forward/reverse propagation, analytical maneuver agreement, conic boundaries, and explicit impact/escape/failure behavior. Position errors and velocity errors are reported separately, in km and km/s; they are never combined into a mixed-unit norm.

An independently written Cartesian acceleration implementation is integrated by SciPy 1.17.1 DOP853. For the universal solver, inclined, eccentric, parabolic and hyperbolic cases require position error below 1e-4 km and velocity error below 1e-7 km/s. All four pass; the largest sampled position error is 1.185e-06 km.



Absolute relative drift is sampled at 201 points over twenty circular periods. Acceptance is below 1e-9 for both energy and angular momentum. Values plotted at 1e-17 include exact numerical zeros.

The independent reference also undergoes a sensitivity check: tighten its tolerances and halve its maximum step. Agreement checks implementation consistency for the selected equations and scenarios. It does not establish that the force model includes every physical effect.

INVESTIGATION DESIGN

A bounded test of Earth's oblateness

Hypothesis: with identical initial states, adding Earth's equatorial-bulge term will produce systematic orbital-plane precession and growing positional separation. The effect should appear sooner at low altitude. A retrograde case should reverse the sign of the node drift.

The comparison uses a fixed Earth pole along +Z and J2 = 1.08262668e-3. The perturbing acceleration is proportional to (3/2) J2 mu R^2 / r^5 times $[x(5z^2/r^2 - 1), y(5z^2/r^2 - 1), z(5z^2/r^2 - 3)]$. Newtonian acceleration is added separately.

Inputs fixed before evaluation

Three initial osculating circular orbits: 400 km at 51.6 degrees inclination; 700 km at 98 degrees; and 35,786 km at 20 degrees. Initial node, periapsis argument and anomaly are zero by convention. Both force models start from identical vectors. Duration is 86,400 seconds, with output every 300 seconds.

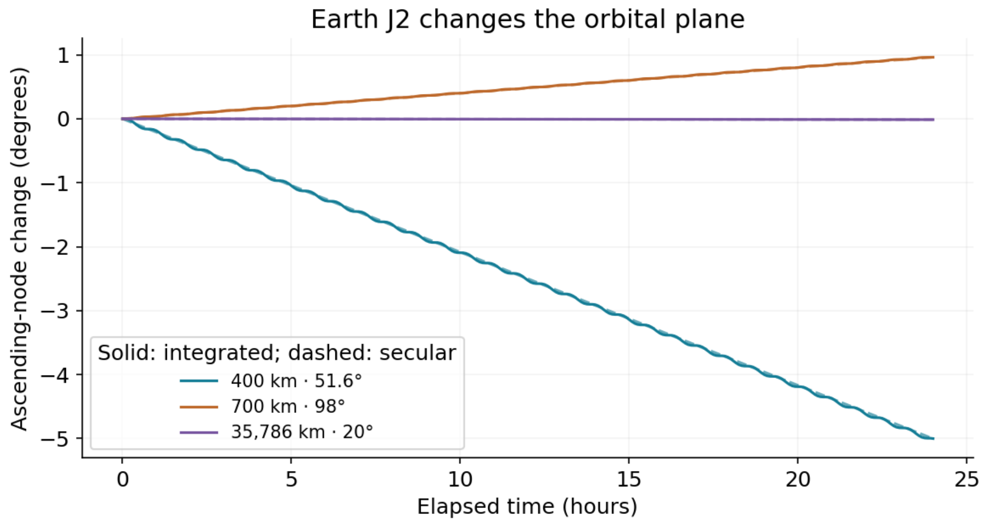
Acceptance criteria

Model adequacy: position separation no greater than 10 km over the desired interval. This is one declared educational tolerance; a different application needs its own requirement. Crossing times are bracketed by adjacent five-minute samples.

J2 numerical solution: maximum reference position error below 0.01 km; velocity error below 1e-5 km/s; 5-vs-10-second step difference below 0.01 km; independent-reference self-sensitivity below 1e-4 km. Conserved total J2 energy and axial angular momentum must drift by less than 1e-8 relatively.

The fitted node rate must agree within 0.1 degree/day with the first-order secular expression: -1.5 J2 n $(R/p)^2$ cos(i). This approximate averaged expression is a trend check, not an exact prediction of instantaneous osculating angles. Full angular momentum is not conserved by the J2 model; its axial component is.

Model differences dominate numerical error



Prograde orbits regress in node; the 98-degree retrograde case advances. Short-period oscillations sit on top of the secular trend.

400 km · 51.6°: first sample over 10 km at 25 minutes (crossing bracket 20-25 minutes). Fitted node rate -5.02485 degrees/day; maximum J2 reference position error 2.366e-05 km; 5-vs-10-second difference 4.812e-04 km.

700 km · 98°: first sample over 10 km at 40 minutes (crossing bracket 35-40 minutes). Fitted node rate +0.96758 degrees/day; maximum J2 reference position error 1.708e-05 km; 5-vs-10-second difference 3.394e-04 km.

35,786 km · 20°: first sample over 10 km at 740 minutes (crossing bracket 735-740 minutes). Fitted node rate -0.01355 degrees/day; maximum J2 reference position error 9.109e-09 km; 5-vs-10-second difference 5.387e-09 km.

All declared numerical checks pass. At low altitude, reducing the step lowers differences strongly. The high-orbit result is near floating-point precision, so its step ratio should not be interpreted as a clean measurement of fourth-order convergence. The downloadable convergence plot shows this scale difference.

The hypothesis is supported for these three scenarios and this tolerance: low orbits diverge sooner, and retrograde node motion reverses sign. This does not establish a universal altitude boundary or a tolerance suitable for every mission.

Make the result inspectable

The reproducibility archive contains TypeScript model and report sources, the independent Python reference generator, pinned dependency requirements, input states, reference samples and exact source hashes. The report records the generating source revision and whether those sources were dirty when generated.

From the archive root: run npm install, then npm run report. To regenerate the independent references, create a Python environment, install scripts/orbit-science-requirements.txt, run python scripts/generate-orbit-reference.py, then npm run report again. Compare numerical fields within their declared tolerances; timestamps and platform-dependent final digits may differ.

What this leaves open

Drag could matter strongly for low Earth orbits. Additional gravity terms, third bodies, real Earth orientation, finite burns and uncertain initial states would change the comparison. Those are future model extensions, not hidden parts of this experiment. A useful next question is how the required force model changes when a positional requirement is tightened or the time horizon is extended.

Sources and public evidence

Project, tour, methods and interactive results
https://dashdahlberg.com/astrophysics/orbital-sandbox/about

Machine-readable report and source hashes
https://dashdahlberg.com/orbit-data/validation/report.json

Independent DOP853 solver documentation
https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.DOP853.html

J2 acceleration implementation reference (Orekit)
https://www.orekit.org/site-orekit-13.1.1/xref/org/orekit/forces/gravity/J2OnlyPerturbation.html

Secular node precession and J2 background (FreeFlyer)
https://ai-solutions.com/_freeflyeruniversityguide/j2_perturbation.htm